A Bifurcation of the Kuramoto Model on Networks

Hayato Chiba (Tohoku Univ. AIMR)

The Kuramoto model is a system of globally coupled phase oscillators for describing synchronization phenomena such as firefly flashing, firing of neurons and circadian muscle cells.

In this talk, the Kuramoto model defined on networks is considered. For the mean-field limit of the model, a bifurcation from the incoherent state to the synchronized state is investigated based on the generalized spectral theory. This reveals that a network topology affects the dynamics through the eigenvalue problem of a certain Fredholm integral operator which defines the structure of a network.

The Riemann hypothesis and holomorphic index in complex dynamics

Tomoki Kawahira (Tokyo Institute of Technology)

We give an interpretation of the Riemann hypothesis in terms of complex and topological dynamics. For example, the Riemann hypothesis is affirmative and all zeros of the Riemann zeta function are simple if and only if a certain meromorphic function has no attracting fixed point. To obtain this, we use holomorphic index (residue fixed point index), which characterizes local properties of fixed points in complex dynamics. We also give some other interpretations and observations by means of Newton's method.

Flocking of Cucker-Smale Models on General Coupling Networks with Free-Will Accelerations

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Phenomena of collective motions such as flocking of birds and schooling of fishes are ubiquitous in the world. Several models have been established to study these. Among them, the one introduced by Cucker and Smale has gained much attention. In this talk, we will discuss the flocking dynamics of the discrete-time Cucker-Smale model under general interaction network topologies with agents having their free-will accelerations. Some theoretical results for the flocking motions would be provided. This is a joint work with Prof. Jonq Juang.

Topologically Mixing Properties of Multiplicative Integer System

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Abstract

Motivated from the study of multiple ergodic average, the investigation of multiplicative shift spaces has caused researcher's interests. This talk focuses on the relations of topologically mixing properties of multiplicative shift spaces and traditional shift spaces. Suppose that X is the multiplicative subshift comes from the shift space Y. Then X is (topologically) transitive/mixing if and only if Y is extensible/mixing. After introducing directional mixing property, we derive the equivalence between directional mixing property of X and weakly mixing property of Y.

Can multiple species coexist in an ecosystem ?

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Competition for resources is a fundamental topic in theoretical ecology. There has been a lot of mathematical models in competition studies. The simplest competition models neglect differences between individuals, using one ordinary differential equation to govern the dynamics of each species. These population dynamics are coupled to dynamics of one or more resources by assuming a constant quota of nutrient per individual, or equivalently, a constant yield of individuals from consumption of a unit of resource. In fact, quotas may vary, leading to variable-internal-stores models.

Ecologists are interested in the mechanism of coexistence/diversity in competitor communities. In this talk, I first review a mathematical model of two species competing in a well mixed chemostat for one resource that is stored internally. For this simple model, two or more species cannot coexist, a result known as the Competitive Exclusion Principle. After introducing additional factors such as multiple resources, toxin mortality, intraguild predation, and spatially/ temporally variations into the model, we find that coexistence or bistability (where outcomes depend on initial conditions) becomes possible.

This talk is based on my recent works joint with Drs. James P. Grover, Sze-Bi Hsu, Jifa Jiang, King-Yeung Lam, Hua Nie, Junping Shi, and Xiaoqiang Zhao.

Keywords: internal storage, coexistence, bistability, intraguild predation, toxin mortality.

On the backward bifurcation of a network-based epidemic model with imperfect vaccination

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We present a study on a network-based susceptible-infected-susceptible epidemic model with a vaccinated compartment. The model incorporates three candidate imperfect vaccines, namely, leaky, all-or-nothing, and waning vaccines. We obtain a threshold value $\mathcal{R}_v$, which determines the stability of a disease-free equilibrium. Furthermore, we perform a bifurcation analysis and conditions that ensure the occurrence of backward bifurcation are derived. More specifically, we show that a stable disease-free equilibrium can coexist with a stable endemic equilibrium when $\mathcal{R}_v < 1$. Numerical experiments are conducted and their results validate the theoretical results.

Keywords: epidemic model, vaccination, equilibria, stability, backward bifurcation.

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TIME-ASYMPTOTIC DYNAMICS OF HERMITIAN RICCATI DIFFERENTIAL EQUATIONS

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The matrix Riccati differential equation (RDE) raises in a wide variety of applications for science and applied mathematics. We are particularly interested in the Hermitian Riccati Differential Equation (HRDE). Radon's Lemma gives a solution representation to HRDE. Although solutions of HRDE may show the finite escape time phenomenon, we can investigate the time asymptotic dynamical behavior of HRDE by its extended solutions. In this paper, we adapt the Hamiltonian Jordan canonical form to characterize the time asymptotic phenomena of the extended solutions for HRDE in four elementary cases. The extended solutions of HRDE exhibit the dynamics of heteroclinic, homoclinic and priodic orbits in the elementary cases under some conditions. This is a joint work with Yueh-Cheng Kuo and Shih-Feng Shieh.

Keywords: Riccati differential equation, Hermitian Riccati differential equation, Radon's Lemma, finite escape time phenomenon, extended solutions, Hamiltonian Jordan canonical form.

The Generalized Riemann Solver of the Multilane Traffic Flow Model

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In this paper we consider a multilane model of traffic flow, which is governed by a hyperbolic system of balance laws. The system of balance laws is given as a 2 by 2 nonlinear hyperbolic system with a discontinuous source term. The global existence of entropy solutions to the Cauchy problem of this multi-lanes model is established by a new version of the generalized Glimm method. The generalized solutions of the Riemann problem, which is the building block of the generalized Glimm scheme, are constructed by Lax's method and an invention of perturbations solving linearized hyperbolic equations with modified source terms. The residuals are estimated for the consistency of the generalized Glimm scheme. The wave interaction estimates are provided for the decay of Glimm functionals and the result of the asymptotic behavior of solutions. This is a joint work with Shih-Wei Chou and John M. Hong.

Keywords: Aw-Rascle Model; multilane model; hyperbolic systems of conservation laws; generalized riemann solver; generalized Glimm scheme; wave interaction.

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Dynamics and Bifurcations of the Hindmarsh-Rose models

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Abstract

In this talk, we first present the bifurcation scenarios of a two-dimensional Hindmarsh–Rose type model with four parameters and simulate some resemblances of neurophysiological features for this model using spike-and-reset conditions. Secondly, we derive requirements for the existence of a Hopf bifurcation in the model and derive equations for the direction and stability of the bifurcation with delay as the bifurcation parameter. When a Hopf bifurcation due to delay occurs, canard-like mixed-mode oscillations are produced at the parameter value for which either the fold bifurcation of cycles or homoclinic bifurcation occurs in the system without delay. Finally, we extend the HR model to one with recurrent neural feedback and spatial information.

Competition for light in forest population dynamics: from computer simulator to mathematical model

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In the presentation I will introduce a size-structured population dynamical model with one and two species for forest growth, and compare this model with a computer forest simulator named SORTIE. The main ingredient taken into account in both models is the competition for light between trees. The parameters of the mathematical model are estimated by fitting to the data generated by SORTIE when the parameter values of SORTIE correspond to the ones previously evaluated for the Great Mountain Forest in USA. We see that the best fit of the parameters of the mathematical model is obtained when the competition for light influences only the growth rate of trees. We also construct a size-structured population dynamical model with spatial structure, and we conduct numerical simulations to observe the spread of trees in space. This work is supervised by Prof. Pierre Magal.

Keywords: computer forest simulator, size-structured model, spatially structured model, state-dependent delay differential equations.

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Problems and Partial Corrections for the article "Quickest Change Detection and Kullback-Leibler Divergence for Two-State Hidden Markov Models"

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Fuh and Mei [1] compute the Kullback-Leibler (KL) divergence of a two-state hidden Markov model (HMM) through the invariant probability measure, characterized by a Fredholm integral equation. The KL divergence is expressed as $J(\theta_1) - J(\theta_0)$, the difference of two upper Lyapunov exponents in their paper. However, there are several subtle problems, in particular the invariant (filter) measure and numerical approximation methods, in their present formulation. We figure out the problems and give corrections for computing $J(\theta_1)$, specifically on the formulas (28) on p.4874, (31-37) on p.4874-4875, and the subsection about numerical methods on p.4875 in [1]. It is quite involved to correct $J(\theta_1)$, and much more involved to correct the other term $J(\theta_0)$ (more accurate $J(\theta_0, \theta_1)$). It needs another paper to study another term (hence the KL divergence). We also elaborate on comparisons of our results and theirs, and show the validity and invalidity of the major results in Fuh and Mei [1].

In addition to corrections, we clarify the concept of filter distribution, the root cause of one class of problems in [1], by regarding it as a second-order distribution. It is known that the HMM filter process is a Markov chain in [2] (and references therein). Denote by Q the transition probability kernel. We derive explicit formulas for Q under various parameter assumptions θ's in theorem 2, and examine Q under the numerical setting θ_1. Under ergodic assumption θ_E, We derive the correct Fredholm integral equation that characterizes the invariant measure in theorem 3, and compute the Lyapunov exponent as an expectation with respect to the invariant measure in theorem 4. In particular, we show an equivalent relation between the Lyapunov exponent for the product of HMM random matrices and the entropy of an HMC, and reveal the close connections between information theory and dynamical systems that have been explored in [3] and [4]. Extensions of our results and methodology are explored in Chen [5], and shall be presented under a more systematic framework and notation system in the coming papers.

Keywords: HMM, filter, invariant measure, Lyapunov exponent, Fredholm integral equation, product of random matrices, entropy rate, dynamical system

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